Problem 1 (Cauchy Mean Value Theorem)

1. Let f, g be continuous on $[a, b]$ and differentiable on (a, b). Show there exists a number $x \in (a, b)$ such that
$$[f(b) - f(a)]g'(x) = [g(b) - g(a)]f'(x).$$
Hint: apply mean value theorem to $h(x) = f(x)[g(b) - g(a)] - g(x)[f(b) - f(a)]$.

2. What happens if f is the identity?

Given two functions f and g differentiable in a neighbourhood around c, **L'Hôpital's Rule** states that if
$$\lim_{x \to c} f(x) = 0 = \lim_{x \to c} g(x),$$
and $\lim_{x \to c} \frac{f'(x)}{g'(x)}$ exists, then
$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}.$$

Problem 2

Consider the following:
$$\lim_{x \to 1} \frac{x^3 - 3x^2 + 4x - 2}{2x^2 - 3x + 1} \overset{\text{L'H}}{=} \lim_{x \to 1} \frac{3x^2 - 6x + 4}{4x - 3} \overset{\text{L'H}}{=} \lim_{x \to 1} \frac{6x - 6}{4} = 0.$$
What is wrong? How can we fix it?

Problem 3

1. Compute $\lim_{x \to 0} \frac{x}{\tan x}$.
2. Compute $\lim_{x \to 0} \cot(x) - x \csc^2(x)$.[a]

[a] $\cot(x) = \frac{\cos(x)}{\sin(x)}$, $\csc(x) = \frac{1}{\sin(x)}$.

Problem 4

If f and g are differentiable and $\lim_{x \to a} \frac{f(x)}{g(x)}$ exists, does it follow that $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ exists?